MATHEMATICS

1. Δ = tanA (tanB. tanC – 1) – 1 (tanC – 1) + 1 (1 – tanB)
= tanA. tanB. tanC – tanA – tanB – tanC + 2 = 2 (as Π tanA = $\Sigma \tan A$)

2. $X^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ $a^2 + bc = 1$ ....(1) $b(a + d) = 0$ ....(2)
$c(a + d) = 0$ ....(3) $bc + d^2 = 1$ ....(4)

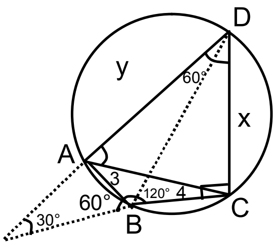
case-I $a + d \neq 0$
$\Rightarrow$ b = 0 and c = 0 $\Rightarrow$ a = ±1 and d = ±1
$\Rightarrow$ (a, d) = (1, 1), (–1, –1) $\Rightarrow$ X = I, –I

case-II a + d = 0
$\Rightarrow$ $a^2 + bc = 1$ $\Rightarrow$ infinite matrices

3. sin x + cos(k + x) + cos(k – x) = 2 $\Rightarrow$ sin x + 2 cos k . cos x = 2

$\therefore$ $2 \le \sqrt{1+4\cos^2 k}$ $\Rightarrow$ $\cos^2 k \ge \frac{3}{4}$ $\Rightarrow$ $k \in \left[n\pi - \frac{\pi}{6}, n\pi + \frac{\pi}{6}\right]$

4.



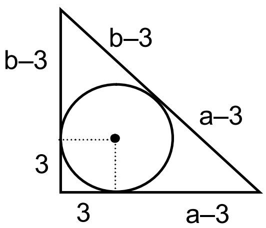
PB = $\frac{3}{\sin 30^\circ}$ = 6 $\therefore$ In ΔPCD $\Rightarrow$ x = 10 tan30° = $\left(\frac{10\sqrt{3}}{3}\right)$

5. cotC = N(cotA + cotB) $\Rightarrow$ $\frac{\cos C}{\sin C} = N\left(\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B}\right)$

$\Rightarrow$ $\frac{a^2+b^2-c^2}{4\Delta} = N\left(\frac{b^2+c^2-a^2}{4\Delta} + \frac{a^2+c^2-b^2}{4\Delta}\right)$ $\Rightarrow$ N = 1007 = 19 × 53

6. Consider n = 2
$\therefore$ $(A^{-1}BA) = (A^{-1}BA).(A^{-1}BA) = A^{-1}B^2A$

7.



$\therefore$ ab = 6s $\Rightarrow$ 2Δ = 6s $\Rightarrow$ r = 3
Now, $a^2 + b^2 = (a + b - 6)^2$ $\Rightarrow$ (a – 6) (b – 6) = 18

8. $AP = \frac{2bc}{b+c}\cos\frac{A}{2} = \frac{4}{3}\ c.\cos\frac{A}{2}$

Now, $9AP^2 + 2a^2 = 16c^2\cos^2\frac{A}{2} + 2a^2 = 16c^2.\ \frac{S(S-a)}{bc} + 2a^2 = 8.\left(\frac{a+3c}{2}\right)\left(\frac{3c-a}{2}\right) + 2a^2 = 18c^2$

9. R.H.S. = $\begin{vmatrix} {}^{x}C_{r} & {}^{x}C_{r+1} & {}^{x}C_{r+2} \\ {}^{y}C_{r} & {}^{y}C_{r+1} & {}^{y}C_{r+2} \\ {}^{z}C_{r} & {}^{z}C_{r+1} & {}^{z}C_{r+2} \end{vmatrix}$

Apply $C_3 \to C_3 + C_2$

$\begin{vmatrix} {}^{x}C_{r} & {}^{x}C_{r+1} & {}^{x+1}C_{r+2} \\ {}^{y}C_{r} & {}^{y}C_{r+1} & {}^{y+1}C_{r+2} \\ {}^{z}C_{r} & {}^{z}C_{r+1} & {}^{z+1}C_{r+2} \end{vmatrix}$

Apply $C_2 \to C_2 + C_1$

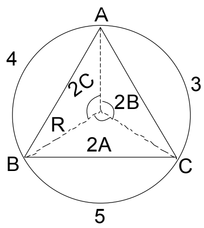
$\begin{vmatrix} {}^{x}C_{r} & {}^{x+1}C_{r+1} & {}^{x+1}C_{r+2} \\ {}^{y}C_{r} & {}^{y+1}C_{r+1} & {}^{y+1}C_{r+2} \\ {}^{z}C_{r} & {}^{z+1}C_{r+1} & {}^{z+1}C_{r+2} \end{vmatrix}$

Apply $C_3 \to C_3 + C_2$

$\begin{vmatrix} {}^{x}C_{r} & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^{y}C_{r} & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^{z}C_{r} & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix}$

10. $\tan^2 x = \frac{1}{3} \quad \Rightarrow \quad x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6} \quad \Rightarrow \quad$ 6 solutions

11.



angle = $\frac{\text{arc}}{\text{radius}}$ ....(1)

$\because\ 4 + 5 + 3 = 2\pi R \quad \Rightarrow \quad R = 6/\pi \qquad \because\ 2A = \frac{5}{R} = \frac{5\pi}{6}$,

$2B = \frac{3}{R} = \frac{\pi}{2}$ and $2C = \frac{4}{R} = \frac{2\pi}{3}$

Area of Δ ABC = $\frac{1}{2}R^2\left[\sin\frac{2\pi}{3} + \sin\frac{5\pi}{6} + \sin\frac{\pi}{2}\right]$

$= \frac{R^2}{2}\left[\frac{\sqrt{3}}{2} + \frac{1}{2} + 1\right] = \frac{R^2}{2}\left[\frac{\sqrt{3}+3}{2}\right] = \frac{\sqrt{3}(\sqrt{3}+1)}{4} \times \frac{36}{\pi^2} = \frac{9\sqrt{3}(\sqrt{3}+1)}{\pi^2}$

12. $\Delta = 0$

$\Rightarrow \begin{vmatrix} \sin 3\theta & -1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{vmatrix} = 0 \quad \Rightarrow \quad \sin\theta = \frac{1}{2}, 0$

13. We have $a^2 + b^2 + c^2 + ab + bc + ca \le 0 \Rightarrow (a + b)^2 + (b + c)^2 + (c + a)^2 \le 0$

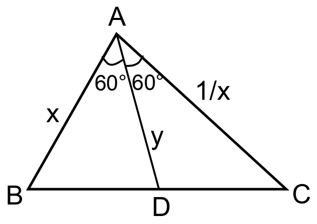
$\therefore$ a + b = 0, b + c = 0, c + a = 0 $\Rightarrow$ a = b= c = 0 $\Rightarrow$ $\Delta = \begin{vmatrix} 4 & 0 & 1 \\ 1 & 4 & 0 \\ 0 & 1 & 4 \end{vmatrix} = 65$

14. $B = A^{2^n} = A^{2 \cdot 2^{n-1}} = (A^2)^{2^{n-1}} = (A^{-1})^{2^{n-1}} = \left(A^{2^{n-1}}\right)^{-1} = \left(A^{2 \cdot 2^{n-2}}\right)^{-1} = \left((A^2)^{2^{n-2}}\right)^{-1}$

$= \left((A^{-1})^{-1}\right)^{2^{n-2}} = A^{2^{(n-2)}} = C \Rightarrow B - C = 0$

15. R.H.S. ≥ 0 for all x, the given condition is true for those values of |x| which lie in the I or III quadrant and the values of x given by B and D satisfy these conditions.

16.



$AD = y = \frac{2bc}{b+c}\cos\frac{A}{2} \Rightarrow y = \frac{1}{x + \frac{1}{x}} \Rightarrow y_{max.} = \frac{1}{2}$

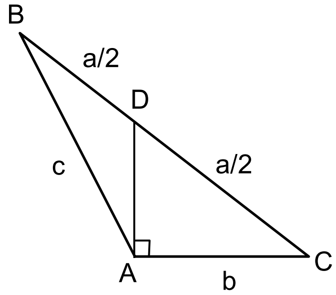
17. $\frac{a}{2\sin A} = R \le \frac{1}{2} \Rightarrow a^2 + b^2 < \frac{1}{4}$ $\therefore$ By A.M. $\ge$ G.M.

$\Rightarrow \frac{a^2+b^2}{2} \ge |ab| \Rightarrow |ab| < \frac{1}{8}$

now, $\frac{a^2+b^2}{2} \ge \left(\frac{a+b}{2}\right)^2$

$(a + b)^2 \le 2(a^2 + b^2) < \frac{1}{2}$

18.



From ΔACD

$\cos C = \frac{2b}{a} \Rightarrow \frac{a^2+b^2-c^2}{2ab} = \frac{2b}{a} \Rightarrow 3b^2 = a^2 - c^2$

Now $\cos A . \cos C = \frac{b^2+c^2-a^2}{2bc}.\frac{2b}{a} = \frac{b^2+c^2-a^2}{ac} = \frac{2(c^2-a^2)}{3ac}$

19. $AB = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \\ 1 \end{bmatrix} \Rightarrow AB = \begin{bmatrix} -3a-7b-5 \\ 2a+4b+3 \\ a+2b+2 \end{bmatrix} = \lambda\begin{bmatrix} a \\ b \\ 1 \end{bmatrix}$

$\Rightarrow \begin{cases} (3+\lambda)a + 7b + 5 = 0 \\ 2a + (4-\lambda)b + 3 = 0 \\ a + 2b + 2 - \lambda = 0 \end{cases} \quad \therefore \begin{vmatrix} 3+\lambda & 7 & 5 \\ 2 & 4-\lambda & 3 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda = 1 \Rightarrow a = -3$ & $b = 1$

20. $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{vmatrix} = 0 \Rightarrow \quad \Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ \alpha & 2 & 4 \\ \alpha^2 & 4 & 10 \end{vmatrix} = 2(\alpha^2 - 3\alpha + 2) = 0$

$\Delta_2 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & 4 \\ 1 & \alpha^2 & 10 \end{vmatrix} = 3(\alpha^2 - 3\alpha + 2) = 0 \Rightarrow \quad \Delta_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \alpha \\ 1 & 4 & \alpha^2 \end{vmatrix} = \alpha^2 - 3\alpha + 2 = 0 \quad \therefore$

$\Rightarrow \quad \alpha = 1, 2$

21. $|M| = \begin{vmatrix} 3 & 4 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & K \end{vmatrix} = -5K$

22. $\begin{vmatrix} a-t & b & c \\ b & c-t & a \\ c & a & b-t \end{vmatrix} = -t^3 + \alpha t^2 + \beta t + \gamma = 0$

product of roots = $\gamma = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

23. AB = A & BA = B

$\Rightarrow \quad AB.A = A^2$ & $BA.B = B^2$

$\Rightarrow \quad A.BA = A^2$ & $B.AB = B^2$

$\Rightarrow \quad AB = A^2$ & $BA = B^2$

$\Rightarrow \quad A = A^2$ & $B = B^2$

$\therefore \quad A^n = A$ & $B^n = B$

Now, $(A^{2015} + B^{2015})^2 = (A + B)^2 = A^2 + B^2 + AB + BA = 2(A + B)$

$(A + B)^3 = 2(A + B)^2 = 4(A + B)$

$(A + B)^4 = 4(A + B)^2 = 8(A + B) \qquad \therefore \qquad (A + B)^n = 2^{n-1}(A + B)$

24. $\Delta = \begin{vmatrix} a^2 + 2^{n+1} + 2p & b^2 + 2^{n+2} + 3q & c^2 + p \\ 2^n + p & 2^{n+1} + q & 2q \\ a^2 + 2^n + p & b^2 + 2^{n+1} + 2q & c^2 - r \end{vmatrix}$

$R_1 \to R_1 - (R_2 + R_3)$

$\Delta = \begin{vmatrix} 0 & 0 & p + r - 2q \\ 2^n + p & 2^{n+1} + q & 2q \\ a^2 + 2^n + p & b^2 + 2^{n+1} + 2q & c^2 - r \end{vmatrix} = 0$

25. $f'(x) = \begin{vmatrix} 2x-5 & 2x-5 & 3 \\ 6x+1 & 6x+1 & 9 \\ 14x-6 & 14x-6 & 21 \end{vmatrix} + \begin{vmatrix} x^2 - 5x + 3 & 2 & 3 \\ 3x^2 + x + 4 & 6 & 9 \\ 7x^2 - 6x + 9 & 14 & 21 \end{vmatrix} = 0$

$\therefore \quad f(x)$ is a constant polynomial & $f(0) \neq 0 \quad \Rightarrow \quad d \neq 0$

26. (A) Replace each element by its cofactor.

(B) $\Delta^2 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} . \begin{vmatrix} a & b & c \\ -c & -a & -b \\ b & c & a \end{vmatrix} = \begin{vmatrix} a^2 & c^2 & 2ac - b^2 \\ 2ab - c^2 & b^2 & a^2 \\ b^2 & 2bc - a^2 & c^2 \end{vmatrix}$

27. $\begin{vmatrix} 1 & a & -1 \\ 2 & -1 & a \\ a & 1 & 2 \end{vmatrix} = 0 \quad \Rightarrow \quad (a+2)(a^2 - 2a - 2) = 0$

28. $A^2 + A + 2I = 0 \Rightarrow A(A + I) = -2I \Rightarrow |A|\ |A + I| = (-2)^n \neq 0 \Rightarrow |A| \neq 0$

29. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$|A + A^T| = \begin{vmatrix} 2a & b+c \\ b+c & 2d \end{vmatrix} = 4ad - (b+c)^2 = 0$

$\Rightarrow \quad \frac{b+c}{2} = \sqrt{ad}$

$\because \quad \frac{b+c}{2} > \sqrt{bc}$

$\Rightarrow \quad \sqrt{ad} > \sqrt{bc} \quad \Rightarrow \quad ad > bc$

$\Rightarrow \quad ad - bc > 0 \quad \Rightarrow \quad |A| > 0$

$|A - A^T| = \begin{vmatrix} 0 & b-c \\ c-b & 0 \end{vmatrix} = (b-c)^2 > 0$

30. Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$|A| = a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2 - a_1 b_3 c_2 - a_2 b_1 c_3 - a_3 b_2 c_1$

$\Rightarrow \quad \det(A) = P_1 + P_2 + P_3 - P_4 - P_5 - P_6 \quad$ where $|P_i| = 1$

$\therefore \quad |\det(A)| \leq |P_1| + |P_2| + |P_3| + |P_4| + |P_5| + |P_6|$

$\Rightarrow \quad |\det(A)| \leq 6$

Hence option (A) is correct.

Now, applying $C_1 \to C_1 + C_2$ & $C_2 \to C_2 + C_3$, we get elements of 1st and 2nd column as even number

$\therefore \quad |A| =$ multiple of 4

Hence option (B) is correct.

31. $8 + a + b = 13 + e + f = 10 + c = 11 + d = k$

$\Rightarrow \quad c = 9, d = 8, (a, b) = (5, 6)$ or $(6, 5)$, $(e, f) = (2, 4)$ or $(4, 2)$

32. $\cos^2\pi x - \sin^2(\pi x - \pi/3) = \frac{1}{2}$

$\Rightarrow \quad \cos^2\pi x - \left(\sin\pi x.\frac{1}{2} - \cos\pi x.\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{2}$

$\Rightarrow \quad \cos^2\pi x - \left(\sin^2\pi x.\frac{1}{4} + \cos^2\pi x.\frac{3}{4} - \frac{\sqrt{3}}{4}\sin 2\pi x\right) = \frac{1}{2}$

$\Rightarrow \quad \frac{1}{4}(\cos^2\pi x - \sin^2\pi x) + \frac{\sqrt{3}}{4}\sin 2\pi x = \frac{1}{2}$

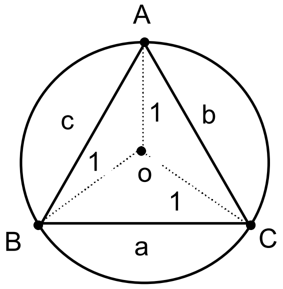
$\Rightarrow \quad \frac{1}{2}\cos 2\pi x + \frac{\sqrt{3}}{2}\sin 2\pi x = 1$

$\Rightarrow \quad \cos\left(2\pi x - \frac{\pi}{3}\right) = 1 \quad \Rightarrow \quad 2\pi x - \frac{\pi}{3} = 2n\pi$

$\Rightarrow \quad x = n + \frac{1}{6} \ ; N \in I$

Sol. **(33 to 35)**

$A = \frac{\pi}{7}, B = \frac{2\pi}{7}, C = \frac{4\pi}{7}$



(33) $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A \cos B \cos C$

$= -1 - 4\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}$

$= -1 - 4\frac{\sin\left(\frac{8\pi}{7}\right)}{8\sin\frac{\pi}{7}} = -\frac{1}{2}$

(34) $\cos 2A + \cos 2B + \cos 2C = -\frac{1}{2}$

$\Rightarrow \quad \frac{1+1-a^2}{2.1.1} + \frac{1+1-b^2}{2.1.1} + \frac{1+1-c^2}{2.1.1} = -\frac{1}{2} \quad \Rightarrow \quad a^2 + b^2 + c^2 = 7$

(35) $\Delta = \frac{1}{2}(\sin 2A + \sin 2B + \sin 2C) = 2\sin A \sin B \sin C$

$= 2.\sin\frac{\pi}{7}\sin\frac{2\pi}{7}\sin\frac{4\pi}{7} = 2\sin\frac{\pi}{7}\sin\frac{2\pi}{7}\sin\frac{3\pi}{7}$

$= 2.\sqrt{\sin^2\frac{\pi}{7}.\sin^2\frac{2\pi}{7}\sin^2\frac{3\pi}{7}} = 2.\sqrt{\frac{7}{2^{7-1}}} = \frac{\sqrt{7}}{4}$ square units

Sol. **(36)** $Q^2 = P'AP.P'AP = P'A^2P$

$\Rightarrow \quad Q^{2015} = P'A^{2015}P$

$\therefore \quad PQ^{2015}P' = PP'A^{2015}PP' = A^{2015} = A^{2014}.A = (A^2)^{1007}.A = (I)^{1007}.A = A$

(37) $PQ^6P' = A^6$

Now, $A^2 = 2A$

$\Rightarrow \quad A^3 = 2A.A = 4A$

$\Rightarrow \quad A^6 = 16A^2 = 32A = 2^5A$

(38) $AA^T = I$

$\Rightarrow \quad 2a^2 = 1, 6b^2 = 1, 3c^2 = 1$

$\Rightarrow \quad 36a^2b^2c^2 = 1$

$\Rightarrow \quad 6|abc| = 1$

39. $f'(x) = 0 \quad \Rightarrow \quad f(x)$ is a constant function $\quad \therefore \quad f(x) = \frac{1}{4}$

40. Here 24 matrices are possible.

Values of determinants can be –8, – 4, – 2, 2, 4, 8

(A) Possible non-negative values of |A| are 2, 4, 8

(B) Sum of these 24 determinants is 0

(C) Mod. (det(A)) is least $\quad \therefore |A| = \pm 2 \quad \Rightarrow |\text{adj}(\text{adj}(\text{adj}(A)))| = |A|^{(n-1)^3} = \pm 2$

(D) Least value of det.(A) is –8 $\quad$ Now $|4A^{-1}| = 16\frac{1}{|A|} = \frac{16}{-8} = -2$